

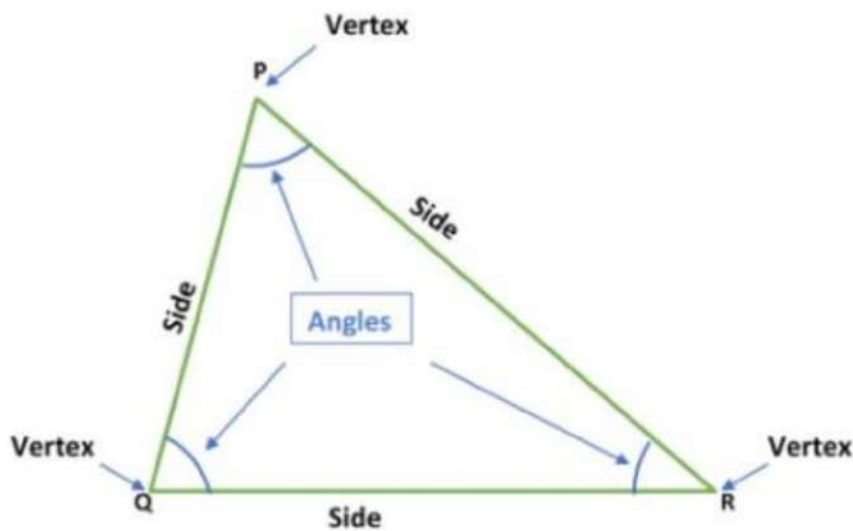
# Chapter – 7

## Triangles

### Congruence of Triangles

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A closed; two-dimensional figure formed by three intersecting lines is called a triangle.



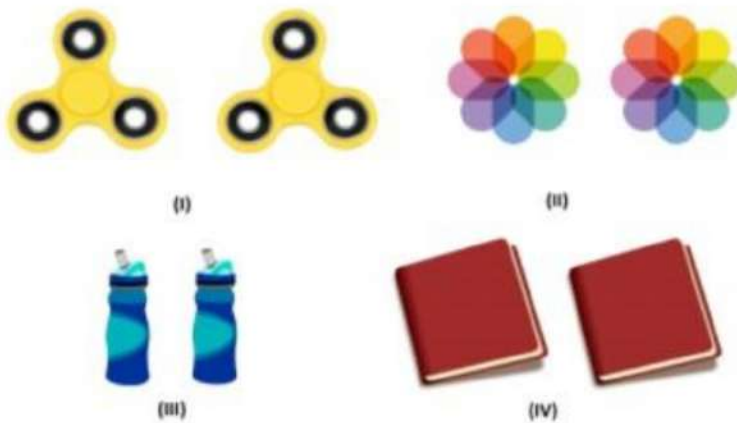
A triangle has three sides, three angles, and three vertices. For example, in the triangle PQR, PQ, QR, and RP are the three sides,  $\angle QPR$ ,  $\angle PQR$ ,  $\angle PRQ$  are the three angles and P, Q and R, are three vertices.

A triangle is a unique figure; it is everywhere around us. We can see the sandwiches in the shape of a triangle, traffic signals, cloth anger, set square, etc. All these are in the shape of a triangle.





Congruency of Shapes/Figures  
Observe the following figures:



In the figures shown above, each pair is identical to each other. Such figures are called congruent (they are similar and fit over one another exactly).

Congruence of Line segments

Any two-line segments are congruent if and only if their lengths are equal. For example, the two-line segments AB and CD are congruent if  $AB = CD$ . Here,  $AB = 3\text{ cm}$  and  $CD = 3\text{ cm}$ .



### Congruence of Angles

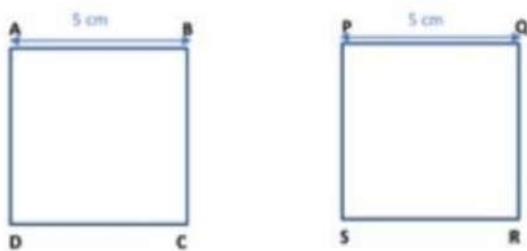
Two angles are congruent if and only if their measures are equal. That is, two angles ABC and PQR are congruent if and only if  $m\angle ABC = m\angle PQR$



Here,  $\angle ABC = 40^\circ$  and  $\angle PQR = 40^\circ$ . So, both angles are congruent.

### Congruence of Squares

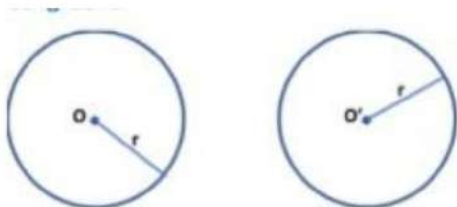
Two squares are congruent if and only if their sides are equal. That is, two squares ABCD and PQRS are congruent if and only if the sides of ABCD is equal to the sides of PQRS.



Here, the sides of both squares are equal to 5 cm. So, these squares are congruent.

### Congruence of Circles

Two circles are congruent if and only if their radius is equal. In the given two circles, both have radius =  $r$ . Hence, the given circles are congruent.

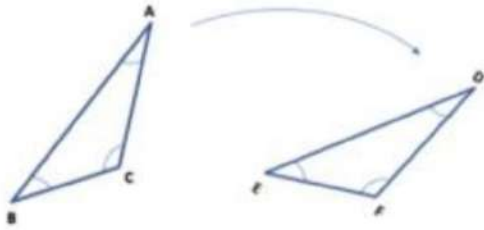


### Congruence of Triangles

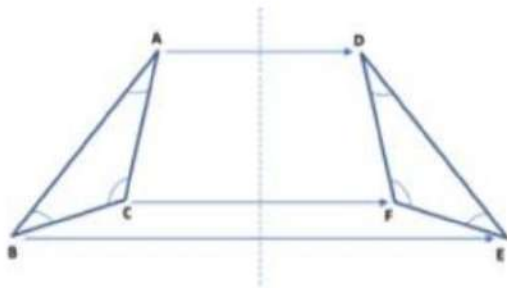
In transformation geometry, we can manipulate shapes in the following 3

ways.

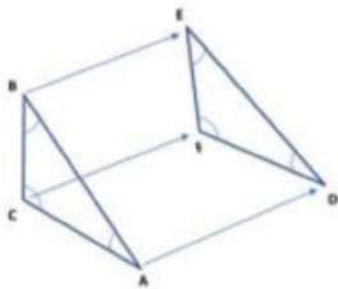
i. Rotation (Turn)



ii. Reflection (Flip)



iii. Translation (Slide)

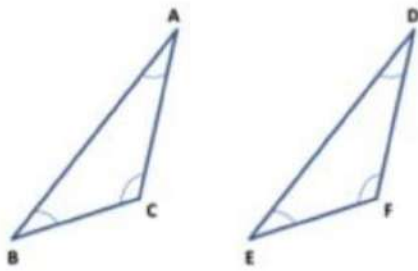


After any of these transformations (turn, flip or slide), the shape still has the same size, area, angles, and line lengths.

If a shape becomes another using the above transformations (turn, slide, and flip) then the two shapes are said to be congruent.

When we superpose  $\Delta ABC$  on  $\Delta DEF$ , such that it covers the other triangle completely & exactly.





In such a superposition the vertices of  $\Delta ABC$  will fall on the vertices of  $\Delta DEF$ , in some order.

The vertex A coincides with vertex D, vertex B coincides with vertex E and vertex C coincides with vertex F.

The side AB coincides with side DE, side AC coincides with side DF and side BC coincides with side EF.

$m\angle BAC = m\angle EDF$ ,  $m\angle ACB = m\angle DFE$  and  $m\angle CBA = m\angle FED$ .

There is a one-on-one correspondence between the vertices of the two congruent triangles. That is, A corresponds to D, C corresponds to F, B corresponds to E and so on

which is written as  $A \leftrightarrow D$ ,  $C \leftrightarrow F$ ,  $B \leftrightarrow E$ .

Under this correspondence  $\Delta ABC \cong \Delta DEF$ ; but it will not be correct to write  $\Delta BCA \cong \Delta DEF$ .

In congruent triangles corresponding parts are equal and we write in short 'CPCT' for corresponding parts of congruent triangles.

Then, we have the following six equalities:

$AB = DE$ ,  $BC = EF$ ,  $AC = DF$  (Corresponding sides are congruent)  $\angle BAC = \angle EDF$ ,  $\angle ACB = \angle DFE$ ,  $\angle CBA = \angle FED$  (Corresponding angles are congruent).

From the above discussion we obtain the following general condition for the congruence of two triangles:

- Triangles are congruent when all corresponding sides and angles are equal.
- The triangles will have equal shape and size.





- Two triangles can be superimposed side to side and angle to angle.
- They have equal area and equal perimeter.
- Congruence is denoted by the symbol  $\cong$ .

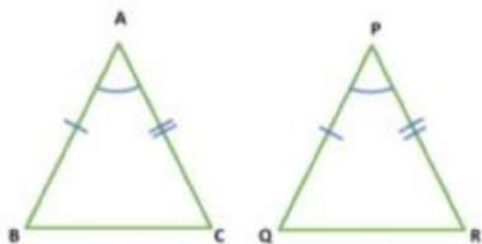
## Criteria for Congruence of Triangles

### Criteria for Congruence of Triangles

**Axiom 1: Side-Angle-Side (SAS) congruence rule**

(An axiom is a mathematical statement which is assumed to be true without proof.)

Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.



Given: Two triangles ABC and PQR such that  $AB = PQ$ ,  $AC = PR$  and  $\angle BAC = \angle QPR$ .

To prove:  $\Delta ABC \cong \Delta PQR$ .

**Proof:** This result cannot be proved with help of previously known results. So, this rule is accepted as an axiom.

Another way to check whether the given two triangles are congruent or not, we follow a practical approach.

Place  $\Delta ABC$  over  $\Delta PQR$  such that the side AB falls on side PQ, vertex A falls on vertex P and B on Q. Since  $\angle BAC = \angle QPR$ . Therefore, AC will fall on PR. But  $AC = PR$  and A

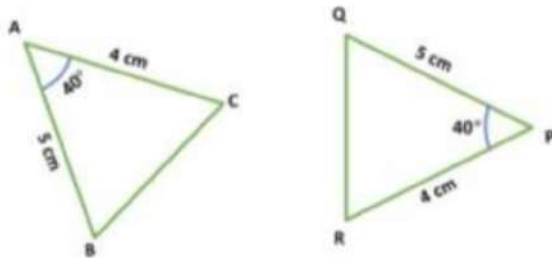
falls on P. therefore, C will fall on R. Thus, AC coincides with PR.

Now, B falls on Q and C falls on R. Therefore, BC coincides with QR.



Thus,  $\Delta ABC$  when superposed on  $\Delta PQR$ , covers it exactly. Hence, by the definition of congruence,  $\Delta ABC \cong \Delta PQR$ .

Example 1: Check whether  $\Delta ABC$  and  $\Delta PQR$  are congruent or not.



Solution: In  $\Delta ABC$  and  $\Delta PQR$ , we have

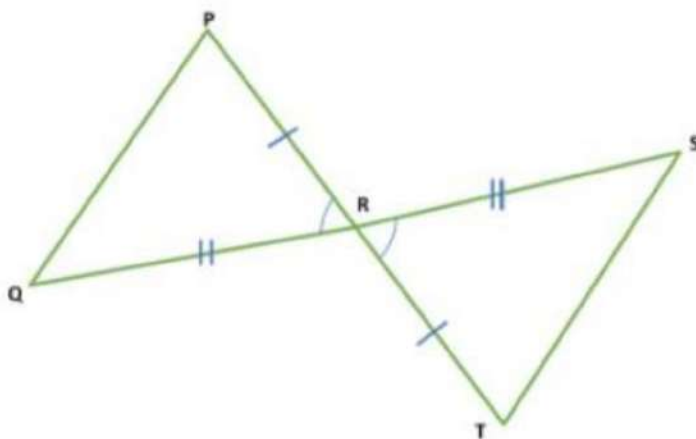
$$AB = PQ = 5 \text{ cm (Given)}$$

$$\angle BAC = \angle QPR = 40^\circ \text{ (Given)}$$

$$AC = PR = 4 \text{ cm (Given)}$$

Therefore,  $\Delta ABC \cong \Delta PQR$  (By SAS-criterion of congruence)

Example 2: In the figure below, R is the mid-point of  $PT$  and  $SQ$ . Prove that  $\Delta PQR \cong \Delta TSR$ .



Given:  $PR = RT$  and  $SR = RQ$ .

To prove:  $\Delta PQR \cong \Delta TSR$ .

Proof: In  $\Delta PQR$  and  $\Delta TSR$ , we have

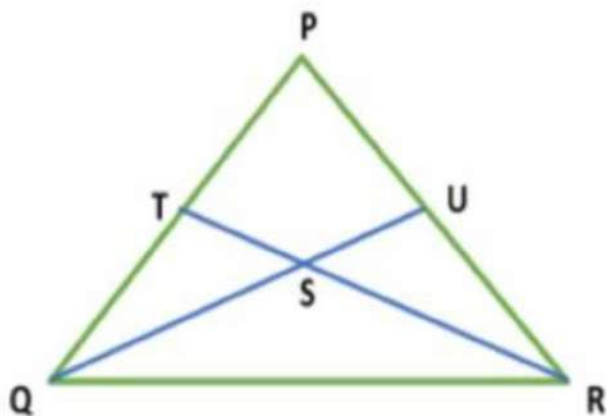
$$PR = TR \quad (\text{R is the mid-point of PT})$$

$$\angle PRQ = \angle TRS \quad (\text{Vertically opposite angles are equal})$$

$$QR = SR \quad (\text{R is the mid-point of SQ})$$

Therefore,  $\Delta PQR \cong \Delta TSR$  (By SAS-criterion of congruence)

Example 3: In the figure, it is given that  $PT = PU$  and  $QT = RU$ . Prove that  $\Delta PTR \cong \Delta PUQ$



Given:  $PT = PU$  and  $QT = RU$ .

To prove:  $\Delta PTR \cong \Delta PUQ$ .

Proof: We have,

$$PT = PU \quad \dots\dots\dots (I)$$

And,  $QT = RU \quad \dots\dots\dots (II)$

Adding equation (I) and (II), We get,

$$PT + QT = PU + RU$$

$$\Rightarrow PQ = PR \quad \dots\dots\dots (III)$$



Now, in  $\Delta PTR$  and  $\Delta PUQ$ , we have

$$PT = PU \text{ [Given]}$$

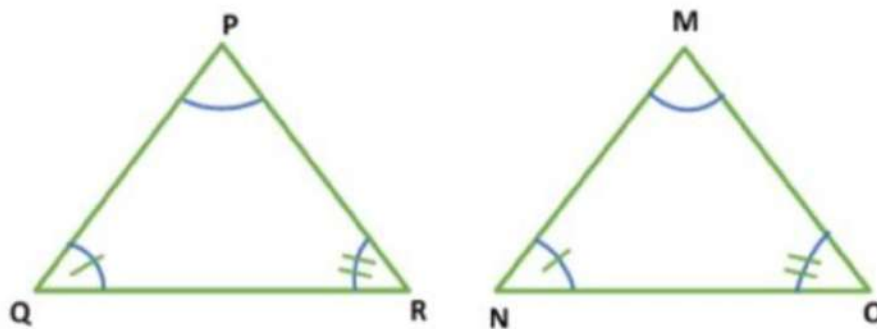
$$\Rightarrow \angle TPR = \angle UPQ \quad \text{[Common]}$$

$$\Rightarrow PQ = PR \quad \text{[From (III)]}$$

Therefore,  $\Delta PTR \cong \Delta PUQ$  [By SAS-criterion of congruence]

**Theorem 1: Angle-Side-Angle (ASA) Congruence rule** Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the

included side of the other triangle.



Given:  $\Delta PQR$  and  $\Delta MNO$  such that  $\angle PQR = \angle MNO$ ,  $\angle PRQ = \angle MON$  and  $QR = NO$ .

To prove:  $\Delta PQR \cong \Delta MNO$ .

Proof: There are three possibilities that arise.

CASE I: When  $PQ = MN$

In this case, we have

$$PQ = MN$$

$$\angle PQR = \angle MNO \quad \text{(Given)}$$

$$QR = NO \quad \text{(Given)}$$

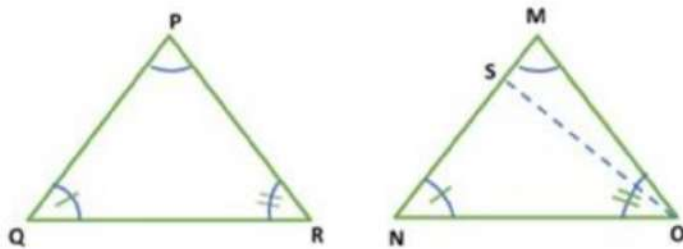
Therefore,  $\Delta PQR \cong \Delta MNO$  (By SAS-criterion of congruence)

CASE II:  $PQ < MN$

Construction: Join OS such that  $NS = PQ$ .

In  $\Delta PQR$  and  $\Delta SNO$ , we have

$$PQ = SN$$



$$\angle PQR = \angle MNO \quad (\text{Given})$$

$$QR = NO \quad (\text{Given})$$

Therefore,  $\Delta PQR \cong \Delta SNO$  (By SAS-criterion of congruence)

By using corresponding parts of congruent triangles

$$\Rightarrow \angle PRQ = \angle SON.$$

But,  $\angle PRQ = \angle MON$ . (Given)

This is possible only when ray SO coincides with ray MO or S coincides with M. Therefore, PQ must be equal to MN.

$$\therefore \angle SON = \angle MON.$$

Thus, in  $\Delta PQR$  and  $\Delta MNO$ , we have

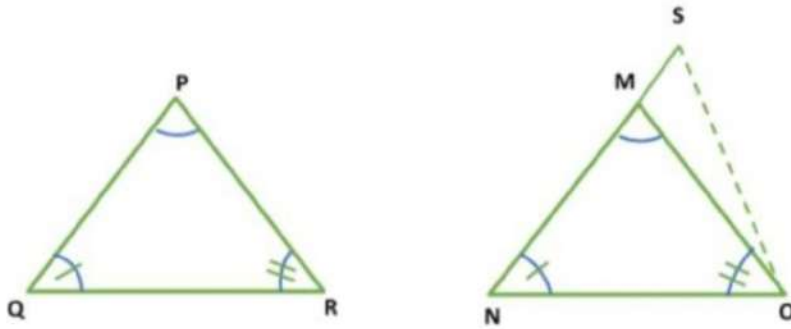
$$PQ = MN$$

$$\Rightarrow \angle PQR = \angle MNO \quad (\text{Given})$$

$$\Rightarrow QR = NO \quad (\text{Given})$$

Therefore,  $\Delta PQR \cong \Delta MNO$  (By SAS-criterion of congruence)

CASE III:  $PQ > MN$ .



Construction: Join SO such that  $NS = PQ$ .

In  $\Delta PQR$  and  $\Delta SNO$ , we have

$$PQ = SN$$

$$\angle PQR = \angle MNO \text{ (Given)}$$

$$QR = NO \text{ (Given)}$$

Therefore,  $\Delta PQR \cong \Delta SNO$  (By SAS-criterion of congruence)

By using corresponding parts of congruent triangles

$$\Rightarrow \angle PRQ = \angle SON.$$

But,  $\angle PRQ = \angle MON.$  (Given)

This is possible only when ray SO coincides with ray MO or S coincides with M.  
Therefore, PQ must be equal to MN.

$$\therefore \angle SON = \angle MON.$$

Thus, in  $\Delta PQR$  and  $\Delta MNO$ , we have

$$PQ = MN$$

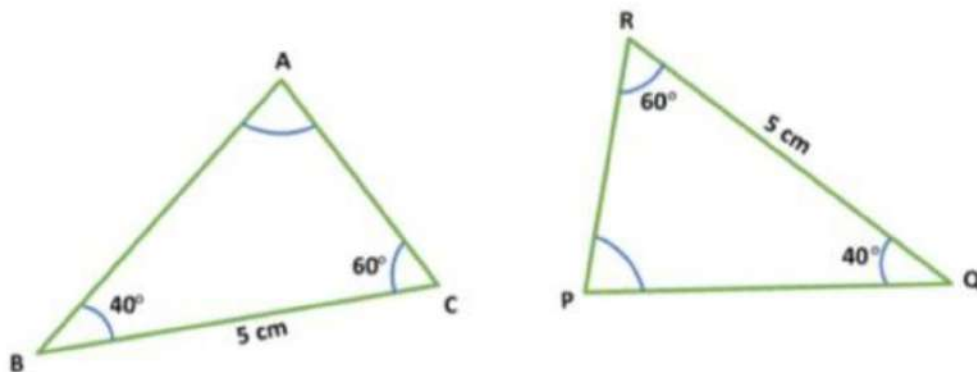
$$\Rightarrow \angle PQR = \angle MNO \text{ (Given)}$$

$$\Rightarrow QR = NO \text{ (Given)}$$

Therefore,  $\Delta PQR \cong \Delta MNO$  (By SAS-criterion of congruence)

Hence, in all the three cases, we have  $\Delta PQR \cong \Delta MNO$ .

Example 1: Check whether  $\triangle ABC \cong \triangle PQR$ ?



Solution: In  $\triangle ABC$  and  $\triangle PQR$ , we have

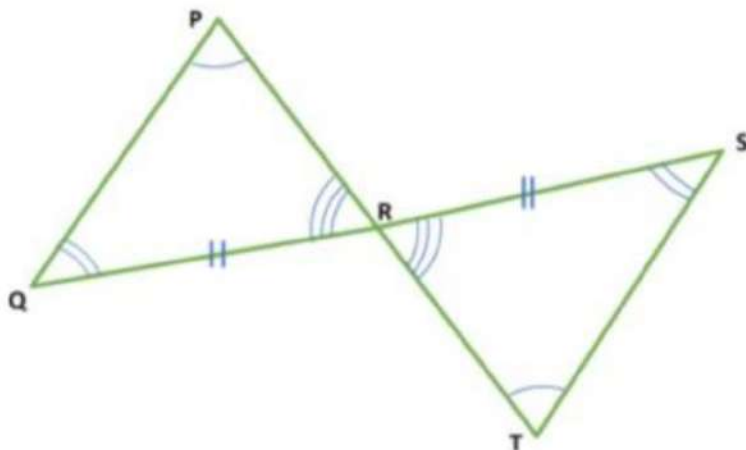
$$\angle ABC = \angle PQR = 40^\circ \quad (\text{Given})$$

$$BC = QR = 5 \text{ cm} \quad (\text{Given})$$

$$\angle BCA = \angle QRP = 60^\circ \quad (\text{Given})$$

Therefore,  $\triangle ABC \cong \triangle PQR$  (By ASA-criterion of congruence)

Example 2: In the figure,  $ST \parallel QP$  and R is the mid-point of SQ, prove that R is also the mid-point of PT.



Given:  $ST \parallel QP$  and R is the mid-point of SQ.

To prove:  $PR = RT$ .

Proof: Since  $QP \parallel ST$  and transversal  $PT$  cuts them at  $P$  and  $T$  respectively.

$\therefore \angle RTS = \angle RPQ$ . (Alternate interior angles)  
..... (I)

Similarly,  $\angle RST = \angle RQP$ . (Alternate interior angles)  
..... (II)

Since  $PT$  and  $SQ$  intersect at  $R$ .

$\therefore \angle QRP = \angle SRT$ . (Vertically opposite angles)  
.....(III)

Thus, in  $\Delta PQR$  and  $\Delta SRT$ , we have

$\angle PQR = \angle RST$  [From (II)]  
 $\Rightarrow QR = RS$  [Given]  
 $\Rightarrow \angle QRP = \angle SRT$  [From (III)]

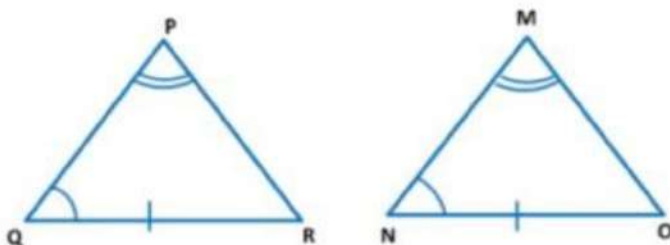
Therefore,  $\Delta PQR \cong \Delta SRT$  [By ASA-criterion of congruence]

By using corresponding parts of congruent triangles

$\Rightarrow PR = RT$ .

Hence,  $R$  is the mid-point of  $PT$ .

**Theorem 2: Angle-Angle-Side (AAS) Congruence rule** If any two angles and a non-included side of one triangle are equal to the corresponding angles and side of another triangle, then the two triangles are congruent.



Given: In  $\Delta PQR$  and  $\Delta MNO$ ,  $\angle PQR = \angle MNO$ ,  $\angle QPR = \angle NMO$  &  $QR = NO$ .



To prove:  $\Delta PQR \cong \Delta MNO$

Proof: We have,

$$\angle PQR = \angle MNO \text{ and } \angle QPR = \angle NMO$$

$$\Rightarrow \angle PQR + \angle QPR = \angle NMO + \angle MNO \quad \dots\dots\dots (I)$$

$$\because \angle PRQ + \angle PQR + \angle QPR = 180^\circ \text{ [Sum of angles of a triangle]}$$

$$\therefore \angle PQR + \angle QPR = 180^\circ - \angle PRQ \quad \dots\dots\dots (II)$$

$$\text{Similarly, } \angle NMO + \angle MNO = 180^\circ - \angle MON \quad \dots\dots\dots (III)$$

From equation (I), (II) & (III), we have

$$180^\circ - \angle PRQ = 180^\circ - \angle MON$$

$$\Rightarrow \angle PRQ = \angle MON \quad \dots\dots\dots (IV)$$

Now, in  $\Delta PQR$  and  $\Delta MNO$ , we have

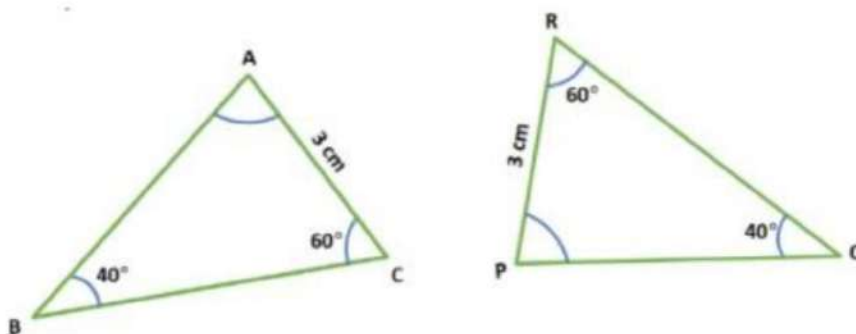
$$\angle PQR = \angle MNO \quad \text{[Given]}$$

$$QR = NO \quad \text{[Given]}$$

$$\angle PRQ = \angle MON \quad \text{[From (IV)]}$$

Therefore,  $\Delta PQR \cong \Delta MNO$  [By ASA-criterion of congruence]

Example1: Check whether two triangles ABC and PQR are congruent.



Solution: In  $\Delta ABC$  and  $\Delta PQR$ , we have

$$\angle ABC = \angle PQR = 40^\circ \quad (\text{Given})$$

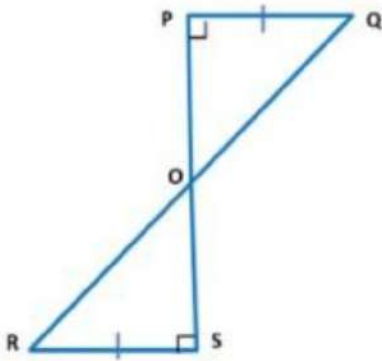
$$\angle BCA = \angle QRP = 60^\circ \quad (\text{Given})$$

$$AC = PR = 3 \text{ cm} \quad (\text{Given})$$

Therefore,  $\Delta ABC \cong \Delta PQR$  (By AAS-criterion of congruence)

Example 2: PQ and RS are perpendiculars of equal length, to a line segment PS.

Show that QR bisects PS.



Given:  $PQ = RS$ ,  $PQ \perp PS$  and  $RS \perp PS$ .

To prove:  $OP = OS$ .

Proof: In triangles OPQ and ORS, we have

$$\angle POQ = \angle SOR \quad [\text{Vertically opposite angles}]$$

$$\angle OPQ = \angle OSR \quad [\text{Each equal to } 90^\circ]$$

$$PQ = RS \quad [\text{Given}]$$

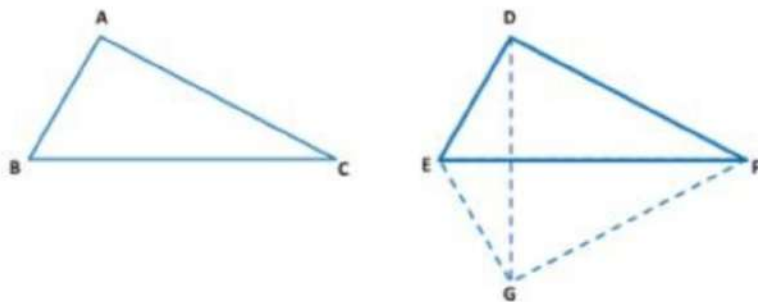
Therefore,  $\Delta POQ \cong \Delta OSR$  [By AAS-criterion of congruence]

By using corresponding parts of congruent triangles

$$\Rightarrow OP = OS.$$

Hence, O is the mid-point of PS.

**Theorem 3: Side-Side-Side (SSS) Congruence rule** If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.



Given that: Two  $\triangle ABC$  and  $\triangle DEF$  such that  $AB = DE$ ,  $BC = EF$  and  $AC = DF$ .

To prove:  $\triangle ABC \cong \triangle DEF$ .

Construction: Suppose BC is the longest side. Draw EG such that  $\angle FEG = \angle ABC$  and  $EG = AB$ . Join GF and GD

Proof: In  $\triangle ABC$  and  $\triangle GEF$ , we have

$BC = EF$	[Given]
$AB = GE$	[By Construction]
$\angle ABC = \angle GEF$	[By Construction]

Therefore,  $\triangle ABC \cong \triangle GEF$  [By SAS-criterion of congruence]

By using corresponding parts of congruent triangles

$$\Rightarrow \angle BAC = \angle EGF \text{ and } AC = GF$$

Now,  $AB = DE$  and  $AB = GE$

$$\Rightarrow DE = GE \quad \dots\dots\dots (1)$$

Similarly,  $AC = DF$  and  $AC = GF$

$$\Rightarrow DF = GF \quad \dots\dots\dots (II)$$

In  $\Delta EGD$ , we have

$$DE = GE \quad \text{[From (I)]}$$

Since angles opposite to equal sides of an isosceles triangle are equal.

$$\Rightarrow \angle EDG = \angle EGD \quad \dots\dots\dots (III)$$

In  $\Delta FGD$ , we have

$$DF = GF \quad \text{[From (II)]}$$

Since angles opposite to equal sides of an isosceles triangle are equal.

$$\Rightarrow \angle FDG = \angle FGD \quad \dots\dots\dots (IV)$$

From (III) and (IV), we have,

$$\angle EDG + \angle FDG = \angle EGD + \angle FGD$$

$$\Rightarrow \angle EDF = \angle EGF$$

$$\text{But, } \angle BAC = \angle EDF \quad \dots\dots\dots (V)$$

In  $\Delta ABC$  and  $\Delta DEF$ , we have

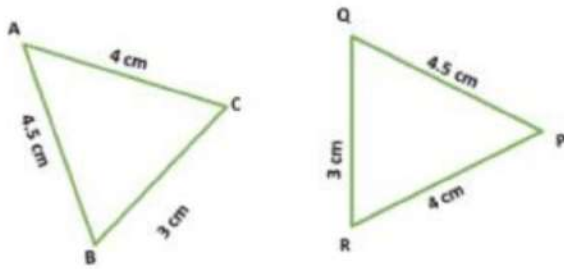
$$AC = DF \quad \text{[Given]}$$

$$\Rightarrow \angle BAC = \angle EDF \quad \text{[From (V)]}$$

$$\text{And, } AB = DE \quad \text{[Given]}$$

Therefore,  $\Delta ABC \cong \Delta DEF$  [By SAS-criterion of congruence]

Example 1: Check whether two triangles ABC and PQR are congruent.

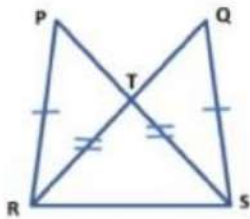


Solution: In  $\Delta ABC$  and  $\Delta PQR$ , we have

$$\begin{aligned} & BC = QR && \text{(Given)} \\ \Rightarrow & AB = PQ && \text{(Given)} \\ \Rightarrow & AC = PR && \text{(Given)} \end{aligned}$$

Therefore,  $\Delta ABC \cong \Delta PQR$  (By SSS-criterion of congruence)

Example 2: In the figure, it is given that  $PR = QS$  and  $PS = RQ$ . Prove that  $\Delta SPR \cong \Delta RQS$ .



Proof : In triangles  $SPR$  and  $RQS$ , we have

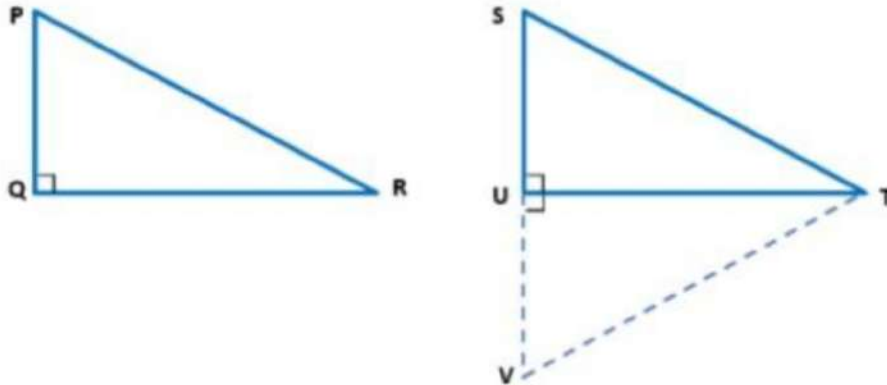
$$\begin{aligned} & PR = QS && \text{[Given]} \\ \Rightarrow & PS = QR && \text{[Given]} \\ \Rightarrow & RS = SR && \text{[Given]} \end{aligned}$$

Therefore,  $\Delta SPR \cong \Delta RQS$  [By SSS-criterion of congruence]

**Theorem 4: Right angle -Hypotenuse-Side (RHS) Congruence rule** If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.







Given that: Two right triangles PQR and SUT in which  $\angle PQR = \angle SU = 90^\circ$ ,  $PR = ST$ ,  $QR = UT$ .

To prove:  $\Delta PQR \cong \Delta SUT$ .

Construction: Produce SU to V so that  $UV = PQ$ . Join VT.

Proof: In  $\Delta PQR$  and  $\Delta SUT$ , we have

$PQ = VU$	[By construction]
$\angle PQR = \angle VUT$	[Each equal to $90^\circ$ ]
$QR = UT$	[Given]

Therefore,  $\Delta PQR \cong \Delta SUT$  [By SAS-criterion of congruence]

By using corresponding parts of congruent triangles

$\Rightarrow \quad \angle QPR = \angle UVT \quad \dots\dots\dots (I)$

And,  $\quad PR = VT$

$\Rightarrow \quad PR = ST \quad \dots\dots\dots (II)$

$\therefore \quad VT = ST$

Since, angles opposite to equal sides in  $\Delta STV$  are equal.

$\Rightarrow \quad \angle UST = \angle TVS \quad \dots\dots\dots (III)$

From (I) and (III), we get,

$$\angle QPR = \angle UST \quad \dots\dots\dots (IV)$$

And given that,  $\angle RQP = \angle TUS \quad \dots\dots\dots (V)$

Adding (IV) & (V), we get,

$$\angle QPR + \angle RQP = \angle UST + \angle TUS \quad \dots\dots\dots (VI)$$

$$\because \angle PRQ + \angle RQP + \angle QPR = 180^\circ$$

$$\therefore \angle QPR + \angle RQP = 180 - \angle PRQ \quad \dots\dots\dots (VII)$$

Similarly,  $\angle UST + \angle TUS = 180^\circ - \angle STU \quad \dots\dots\dots (VIII)$

From equation (VI), (VII) & (VIII), we have

$$180^\circ - \angle PRQ = 180^\circ - \angle STU$$

$$\Rightarrow \angle PRQ = \angle STU. \quad \dots\dots\dots (IX)$$

Now, in  $\Delta PQR$  and  $\Delta SUT$

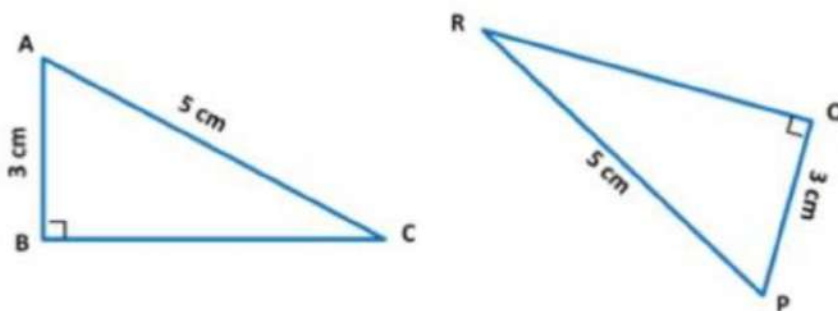
$$QR = UT \quad [Given]$$

$$\Rightarrow \angle PRQ = \angle STU. \quad [From (IX)]$$

And,  $PR = ST \quad [Given]$

Therefore,  $\Delta PQR \cong \Delta SUT$  [By SAS-criterion of congruence]

Example 1: Check whether two triangles ABC and PQR are congruent.



Solution: In  $\Delta ABC$  and  $\Delta PQR$ , we have

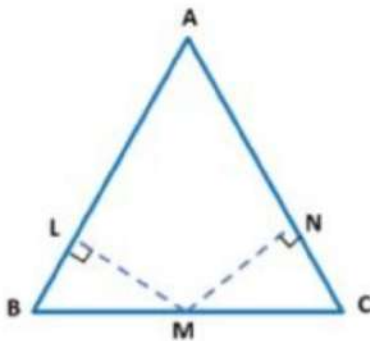
$$\angle ABC = \angle PQR = 90^\circ \quad (\text{Given})$$

$$AC = PR \quad (\text{Given})$$

$$AB = PQ \quad (\text{Given})$$

Therefore,  $\Delta ABC \cong \Delta PQR$  (By RHS-criterion of congruence)

Example 2: In the figure below, it is given that  $LM = MN$ ,  $BM = MC$ ,  $ML \perp AB$  and  $MN \perp AC$ . Prove that  $AB = AC$ .



Proof: In right-angled  $\Delta BLM$  and  $\Delta CNM$ , we have

$$BM = MC \quad [\text{Given}]$$

$$\Rightarrow LM = MN \quad [\text{Given}]$$

Therefore,  $\Delta BLM \cong \Delta CNM$  [By RHS-criterion of congruence]

By using corresponding parts of congruent triangles

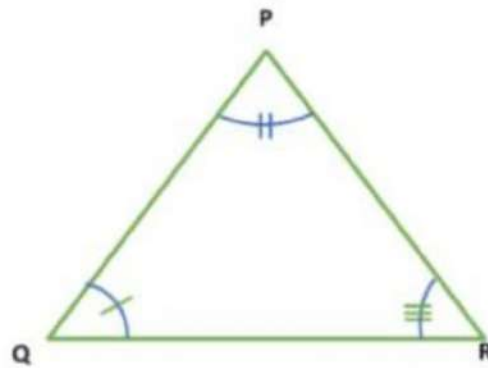
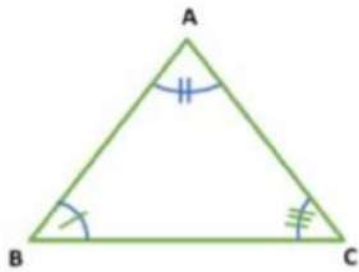
$$\Rightarrow \angle LBM = \angle NCM$$

Since, sides opposite to equal angles are equal.

Therefore,  $AB = AC$ .

Some more rules

### 1. Angle-Angle-Angle (AAA) Rule

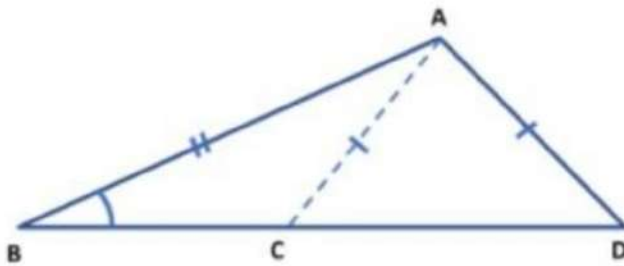


In  $\Delta ABC$  and  $\Delta PQR$ , we have

$$\begin{aligned} \angle BAC &= \angle QPR && \text{(Given)} \\ \angle ACB &= \angle PRQ && \text{(Given)} \\ \angle CBA &= \angle RQP && \text{(Given)} \end{aligned}$$

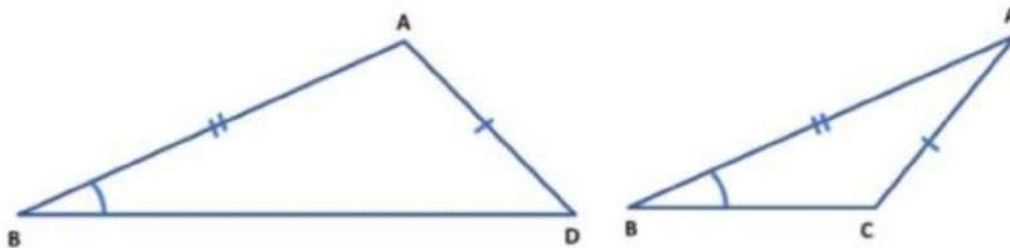
But,  $\Delta ABC$  and  $\Delta PQR$  are similar but not congruent because their sizes are different.

## 2. Angle-Side-Side (ASS or SSA) Rule



We have a triangle  $ABD$ . We draw  $AC$  such that  $AC = AD$ .

Now, consider the two triangles,  $\Delta ABD$  and  $\Delta ABC$



Now, let us check whether we can use ASS criteria for the

congruency of two triangles or not. In  $\Delta ABD$  and  $\Delta ABC$ , we have

$$\begin{array}{ll} \angle ABD = \angle ABC & \text{(Given)} \\ AB = AB & \text{(Common)} \\ AD = AC & \text{(Given)} \end{array}$$

So, the corresponding Angle-Side-Side of the two triangles are equal but, these two figures are different in shape.

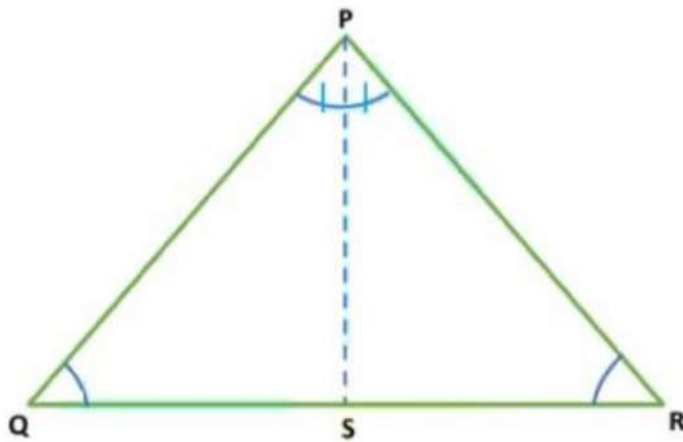
So, we can conclude that this method is not a universal method for proving triangles congruent.

### Some Properties of a Triangle

Some Properties of a Triangle

Theorem 5: Angles opposite to equal sides of an isosceles triangle are equal.

Given:  $\Delta PQR$  is an isosceles triangle in which  $PQ = PR$ .



To prove:  $\angle PQS = \angle PRS$ .

Construction: Draw the bisector  $PS$  of  $\angle QPR$  which meets  $QR$  in  $S$ .

Proof: In  $\Delta PQS$  and  $\Delta PRS$ , we have

$$\begin{array}{ll} PQ = PR & \text{(Given)} \\ \Rightarrow \angle QPS = \angle RPS & \text{(By construction)} \end{array}$$



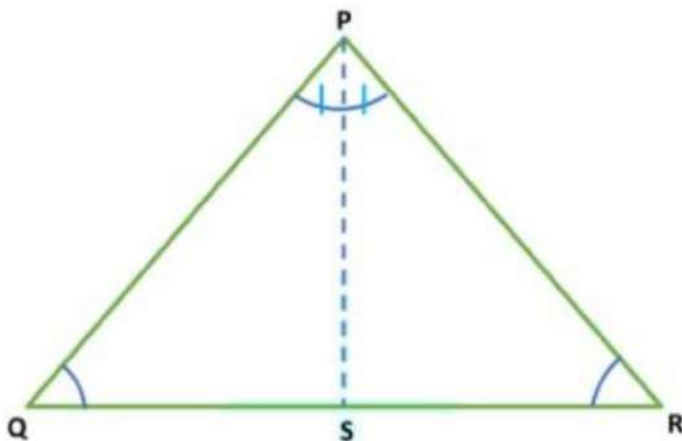
$$\Rightarrow \quad PS = PS \quad (\text{Common})$$

Therefore,  $\Delta PQS \cong \Delta PRS$  (By SAS-criterion of congruence)

By using corresponding parts of congruent triangles

$$\Rightarrow \quad \angle PQS = \angle PRS.$$

Theorem 6: The sides opposite to equal angles of a triangle are equal.



Given: In triangle PQR,  $\angle PQR = \angle PRQ$ .

To prove:  $PQ = PR$ .

Construction: Draw the bisector of  $\angle QPR$  and let it meet QR at S.

Proof: In  $\Delta PQS$  and  $\Delta PRS$ , we have

$$\angle PQR = \angle PRQ \quad (\text{Given})$$

$$\Rightarrow \quad \angle QPS = \angle RPS \quad (\text{By construction})$$

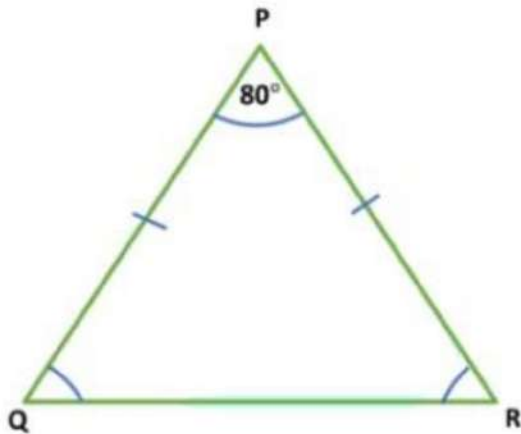
$$\Rightarrow \quad PS = PS \quad (\text{Common})$$

Therefore,  $\Delta PQS \cong \Delta PRS$  ( By AAS-criterion of congruence)

By using corresponding parts of congruent triangles

$$\Rightarrow \quad PQ = PR.$$

Example 1: In  $\Delta PQR$ ,  $\angle QPR = 80^\circ$  and  $PQ = PR$ . Find  $\angle RQP$  and  $\angle PRQ$ .



Given that:  $\angle QPR = 80^\circ$  and  $PQ = PR$ .

Solution: We have

$$PQ = PR$$

Since angles opposite to equal sides are equal.

$$\Rightarrow \angle RQP = \angle PRQ$$

In  $\Delta PQR$ , We have

$$\angle QPR + \angle RQP + \angle PRQ = 180^\circ.$$

$$\Rightarrow \angle QPR + \angle RQP + \angle RQP = 180^\circ. (\because \angle RQP = \angle PRQ)$$

$$\Rightarrow 80^\circ + 2 \angle RQP = 180^\circ$$

$$\Rightarrow 2 \angle RQP = 180^\circ - 80^\circ$$

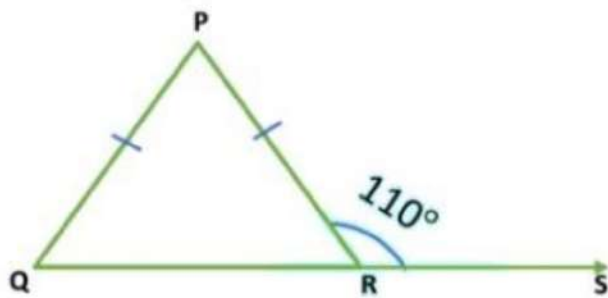
$$\Rightarrow 2 \angle RQP = 100^\circ$$

$$\Rightarrow \angle RQP = \frac{100^\circ}{2}$$

$$\Rightarrow \angle RQP = 50^\circ$$

Hence,  $\angle RQP = \angle PRQ = 50^\circ$

Example 2: In figure,  $PQ = PR$  and  $\angle PRS = 110^\circ$ . Find  $\angle P$ .



Solution: We have,

$$PQ = PR$$

Since angles opposite to equal sides are equal.

$$\Rightarrow \angle PQR = \angle PRQ.$$

Now,  $\angle PRQ + \angle PRS = 180^\circ$  ( $\because$  Linear pair of angles)

$$\Rightarrow \angle PRQ + 110^\circ = 180^\circ$$

$$\Rightarrow \angle PRQ = 180^\circ - 110^\circ$$

$$\Rightarrow \angle PRQ = 70^\circ$$

And also,  $\angle PQR = 70^\circ$ .

Now,  $\angle QPR + \angle PQR + \angle PRQ = 180^\circ$

$$\Rightarrow \angle QPR + 70^\circ + 70^\circ = 180^\circ$$
 ( $\because \angle PQR = \angle PRQ$ )

$$\Rightarrow \angle QPR + 140^\circ = 180^\circ$$

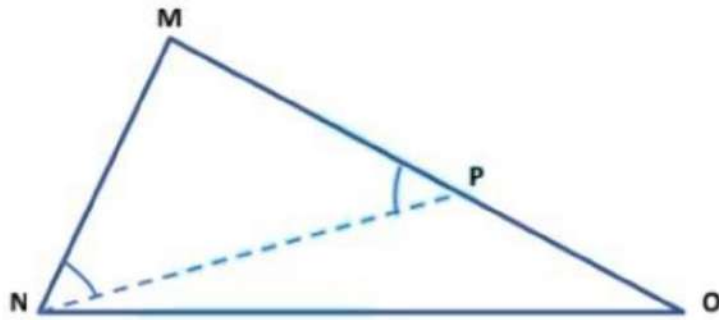
$$\Rightarrow \angle QPR = 180^\circ - 140^\circ$$

$$\Rightarrow \angle QPR = 40^\circ.$$

### Inequalities in a Triangle

Inequalities in a Triangle

**Theorem 7:** If two sides of a triangle are unequal, the angle opposite to the longer side is larger.



Given that: In  $\Delta MNO$ ,  $MO > MN$ .

To prove:  $\angle MNO > \angle MON$ .

Construction: Mark a point P on MO such that  $MN = MP$ . Join NP.

Proof: In  $\Delta MNP$ , we have

$$MN = MP \quad \text{[By construction]}$$

Since angles opposite to equal sides are equal.

$$\Rightarrow \angle MNP = \angle MPN. \quad \dots\dots\dots (I)$$

Now, consider  $\Delta NPO$ . We find that  $\angle MPN$  is the exterior angle of  $\Delta NPO$  and an exterior angle is always greater than interior opposite angles.

Therefore,  $\angle MPN > \angle PON$

$$\Rightarrow \angle MPN > \angle MON \quad \dots\dots\dots (II)$$

From (I) and (II), we have

$$\angle MNP > \angle MON \quad \dots\dots\dots (III)$$

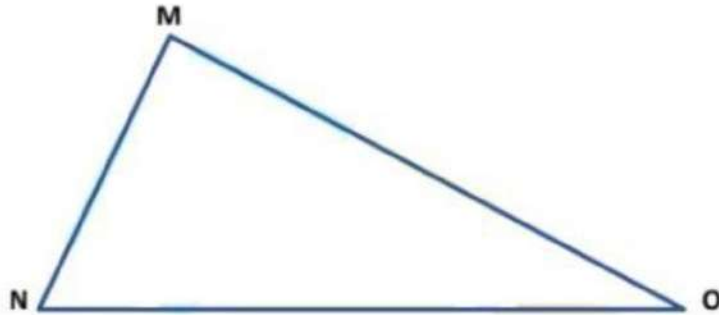
But,  $\angle MPN$  is a part of  $\angle MNO$ .

$$\therefore \angle MNO > \angle MNP \quad \dots\dots\dots (IV)$$

From (III) and (IV), we get

$$\angle MNO > \angle MON.$$

Theorem 8: In any triangle, the side opposite to the larger (greater) angle is longer.



Given that: In  $\Delta MNO$ ,  $\angle MNO > \angle MON$ .

To prove:  $MO > MN$ .

Proof: In  $\Delta MNO$ , we have the following three possibilities.

(I)  $MO = MN$

(II)  $MO < MN$

(III)  $MO > MN$

Out of these three possibilities exactly one must be true.

CASE I: When  $MO = MN$

Since angles opposite to equal sides are equal.

$$\Rightarrow \angle MNO = \angle MON$$

This is a contradiction that  $\angle MNO > \angle MON$ .

$\therefore MO \neq MN$ .

CASE II: When  $MO < MN$

Since the longer side has a greater angle opposite to it.

$$\Rightarrow \angle MNO < \angle MON$$



This also contradicts the given hypothesis that  $\angle MNO > \angle MON$

Thus, we are left with the only possibilities,  $MO > MN$ , which must be true.

Hence,  $MO > MN$ .

Example: In a  $\Delta MNO$ , if  $\angle MON = 55^\circ$  and  $\angle ONM = 66^\circ$ , determine the shortest and largest sides of the triangle.

Solution: We have,

$$\angle MON = 55^\circ \text{ and } \angle ONM = 66^\circ$$

$$\therefore \angle MON + \angle ONM + \angle NMO = 180^\circ$$

$$\Rightarrow 55^\circ + 66^\circ + \angle NMO = 180^\circ$$

$$\Rightarrow 121^\circ + \angle NMO = 180^\circ$$

$$\Rightarrow \angle NMO = 180^\circ - 121^\circ$$

$$\Rightarrow \angle NMO = 59^\circ.$$

Since  $\angle NMO$  and  $\angle ONM$  are the smallest and largest angles respectively. Therefore, sides  $ON$  and  $OM$  are the smallest and largest sides respectively of the triangle.

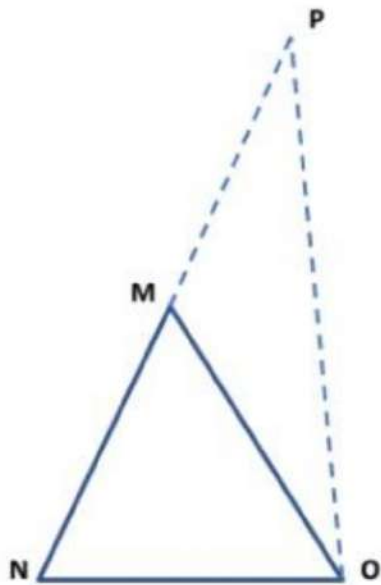
Theorem 9: The sum of any two sides of a triangle is greater than the third side.

Given:  $MNO$  is a triangle.

To prove: (I)  $MN + MO > NO$

(II)  $MN + NO > MO$

(III)  $NO + MO > MN$



Proof: (I) Produce side NM to P such that  $MP = MO$ . Join PO

In  $\Delta MOP$ , we have

$$MP = MO \quad (\text{By construction})$$

Since angles opposite to equal sides are equal.

$$\Rightarrow \angle MOP = \angle MPO$$

$$\Rightarrow \angle NOM + \angle MOP > \angle MPO$$

$$\Rightarrow \angle NOP > \angle MPO$$

Since side opposite to greater angle is larger.

$$\therefore NP > NO$$

$$\Rightarrow MN + MP > NO$$

$$\Rightarrow MN + MO > NO \quad [\because MO = MP \text{ by construction}]$$

Thus,  $MN + MO > NO$ .

(II) Produce side MN to P such that  $NP = NO$ . Join PO.

In  $\triangle NOP$ , we have

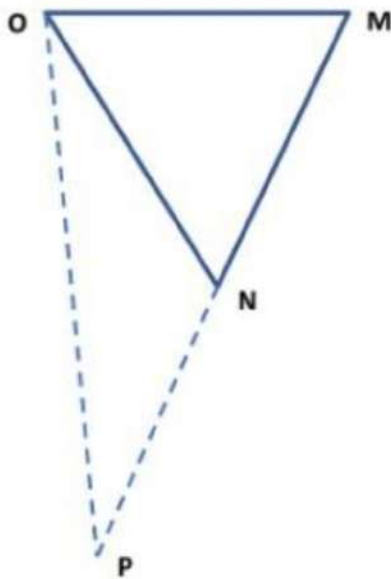
$$NP = NO \quad (\text{By construction})$$

Since angles opposite to equal sides are equal.

$$\Rightarrow \angle NOP = \angle NPO$$

$$\Rightarrow \angle NOM + \angle NOP > \angle NPO$$

$$\Rightarrow \angle MOP > \angle NPO$$



Since side opposite to greater angle is larger.

$$\therefore MP > MO$$

$$\Rightarrow MN + NP > MO$$

$$\Rightarrow MN + NO > MO \quad [\because NO = NP \text{ by construction}]$$

Thus,  $MN + NO > MO$ .

(III) Produce side MO to P such that  $OP = NO$ . Join NP

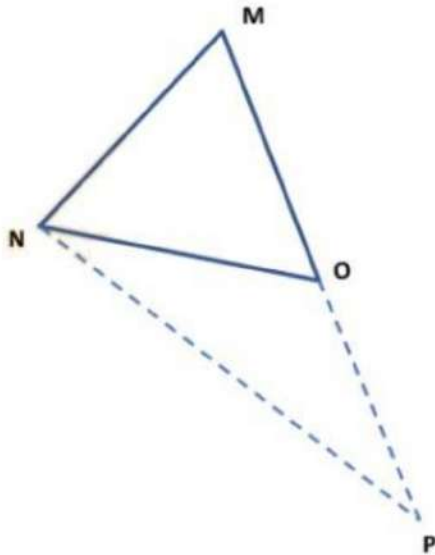
In  $\triangle NOP$ , we have

$$OP = ON \quad (\text{By construction})$$

Since angles opposite to equal sides are equal.



- $\Rightarrow \angle ONP = \angle OPN$
- $\Rightarrow \angle ONM + \angle ONP > \angle OPN$
- $\Rightarrow \angle MNP > \angle OPN$

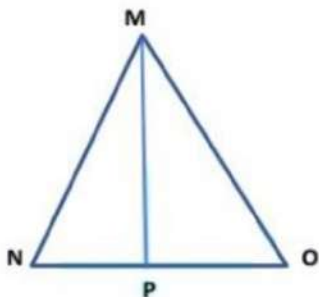


Since, side opposite to greater angle is larger.

- $\therefore MP > MN$
- $\Rightarrow MO + OP > MN$
- $\Rightarrow MO + NO > MN$  [ $\because NO = OP$  by construction]
- Thus,  $MO + NO > MN$ .

Hence,  $MN + MO > NO$ ,  $MN + NO > MO$  and  $NO + MO > MN$ .

Example: In  $\Delta MNO$ , P is any point on the side NO. Show that  $MN + MO + NO > 2 MP$ .



Solution: In  $\Delta MNP$ , we have

$$MN + NP > MP \quad \text{..... (I)}$$

(∵ Sum of two sides of a triangle is greater than the third side)

Similarly, In  $\Delta MOP$ , we have

$$MO + OP > MP \quad \text{..... (II)}$$

Adding (I) and (II), we get

$$(MN + NP) + (MO + OP) > MP + MP$$

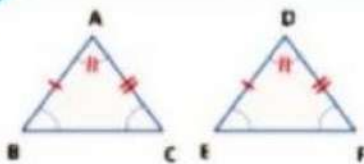
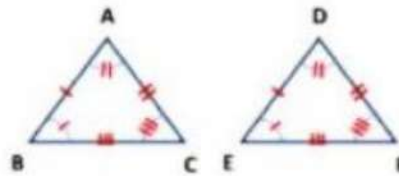
$$\Rightarrow MN + (NP + OP) + MO > 2 MP$$

$$\Rightarrow MN + NO + MO > 2 MP.$$

### Summary - Triangles

Two figures are congruent, if they are of the same shape and of the same size.

Two triangles  $ABC$  and  $DEF$  are congruent under the correspondence  $A \leftrightarrow D, B \leftrightarrow E$  and  $C \leftrightarrow F$ , then symbolically, it is expressed as  $\Delta ABC \cong \Delta DEF$ .  $AB = DE, AC = DF, BC = EF$  &  $\angle BAC = \angle EDF, \angle CBA = \angle FED, \angle ACB = \angle DFE$



**Side-Angle-Side (SAS) congruence rule**

Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.

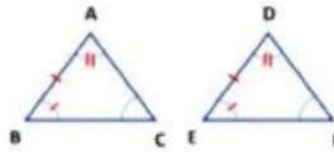
$$AB = DE, \angle BAD = \angle EDF \text{ \& } AC = DF.$$





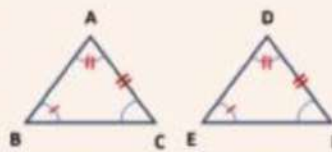
### Angle-Side-Angle (ASA) Congruence rule

Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of another triangle.  $\angle ABC = \angle DEF$ ,  $AB = DE$  &  $\angle BAC = \angle EDF$ .



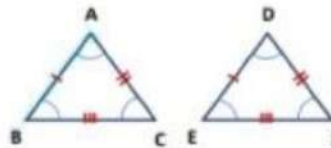
### Angle-Angle-Side (AAS) Congruence rule

If any two angles and a non-included side of one triangle are equal to the corresponding angles and side of another triangle, then the two triangles are congruent.



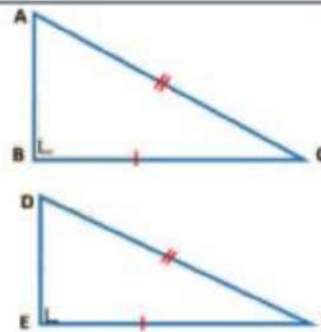
### Side-Side-Side (SSS) Congruence rule

If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.  $AB = DE$ ,  $AC = DF$  &  $BC = EF$ .



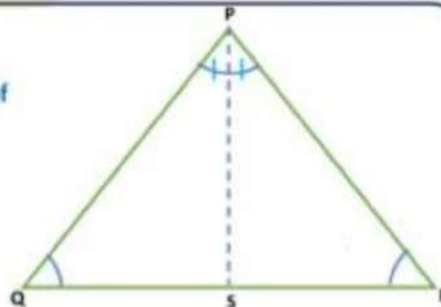
### Right angle – Hypotenuse – Side (RHS) Congruence rule

If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.  $\angle ABC = \angle DEF$ ,  $BC = EF$  &  $AC = DF$ .



### Properties of a Triangle

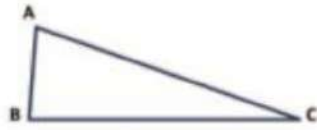
Angles opposite to equal sides of an isosceles triangle are equal.  $\angle PQR = \angle PRQ$   
The sides opposite to equal angles of a triangle are equal.  $PQ = PR$



### Inequalities in a Triangle

If two sides of triangle ABC are unequal, the angle opposite to the longer side is larger.  $\angle ABC > \angle ACB$ .

In triangle ABC, the side opposite to the larger (greater) angle is longer.  $AC > AB$ .



The sum of any two sides of triangle ABC is greater than the third side.  $AB + AC > BC$ ,  $AC + BC > AB$  &  $BC + AB > AC$ .

